Theoretical model for calculation of helicity in solar active regions

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Abstract. We (Choudhuri, Chatterjee and Nandy, 2005) calculate helicities of solar active regions based on the idea of Choudhuri (2003) that poloidal flux lines get wrapped around a toroidal flux tube rising through the convection zone, thereby giving rise to the helicity. Rough estimates based on this idea compare favourably with the observed magnitude of helicity. We use our solar dynamo model based on the Babcock--Leighton α-effect to study how helicity varies with latitude and time. At the time of solar maximum, our theoretical model gives negative helicity in the northern hemisphere and positive helicity in the south, in accordance with observed hemispheric trends. However, we find that, during a short interval at the beginning of a cycle, helicities tend to be opposite of the preferred hemispheric trends. Next we (Chatterjee, Choudhuri and Petrovay 2006) use the above idea along with the sunspot decay model of Petrovay and Moreno-Insertis, (1997) to estimate the distribution of helicity inside a flux tube as it keeps collecting more azimuthal flux during its rise through the convection zone and as turbulent diffusion keeps acting on it. By varying parameters over reasonable ranges in our simple 1-d model, we find that the azimuthal flux penetrates the flux tube to some extent instead of being confined to a narrow sheath outside.

Index Terms. Helicity, solar active regions.

1. Introduction

Solar active regions are known to have helicity associated with them. Observational studies based on vector magnetograms (Pevtsov et al., 1995; 2001; Bao and Zhang, 1998) indicate that the preferred sign of this helicity is opposite in the two hemispheres (negative in northern hemisphere and positive in the southern), in spite of a very large statistical scatter. Fig. 2 of Canfield and Pevtsov (2000) is a typical plot showing a variation of helicity with latitude, which any theoretical model has to explain.

Solar magnetic fields are believed to be produced by the dynamo process. One possibility is that the dynamo process itself is responsible for the generation of helicity. The other possibility is that the rising flux tubes, which eventually form active regions, get twisted by interacting with the helical turbulence in the surrounding convection christened the Σ-effect by Longcope et al. (1998). The two possibilities mentioned above need not be mutually exclusive: both may be simultaneously operative. A careful comparison between observational data and detailed theoretical models will be needed to ascertain the relative importance of these two effects.

In section 2.1, 2 we present calculations of helicity based on our two-dimensional kinematic solar dynamo model presented in Nandy and Choudhuri (2002) and Chatterjee et al. (2004).

In a Babcock-Leighton dynamo poloidal field A is created from the decay of tilted active regions, the amount of tilt being given by the Joy's Law. Also, in accordance with the Hale's polarity rule, a bipolar active region formed by a flux tube with positive B_z would have the leading spot towards the equator than the following spot. Decay of such a pair would thus mean clockwise lines of B_p around active regions. When a new toroidal flux tube with positive B_z moves upwards near the surface, the poloidal field gets wrapped around the toroidal flux tube. Due to high magnetic Reynolds number the flux tubes are not able to cut through the poloidal field lines thus giving rise to the helicity. Using the above idea in section 3.1, we (Chatterjee, Choudhuri and Petrovay, 2006) formulate a 1-d problem to estimate the distribution of helicity in the solar active regions.

2. Estimating the value of Helicity

For, force free fields in the photosphere and the corona we may define helicity as

$$\alpha = \nabla \times B_z / B_z \tag{1}$$

where z corresponds to the vertical direction, which is along the axis of the flux tube for active regions on the surface. The parameter α (not to be confused with the dynamo α-effect traditionally associated with the poloidal field generation mechanism) is a measure of the handedness or chirality of the magnetic field. The typical observed value of the twist parameter α from magnetogram data, calculated by several authors is about 2×10^{-8}/m.

To estimate the value of helicity theoretically, we have to keep in mind that the flux of poloidal field B_P through the whole solar convection zone (SCZ) gets dragged by the toroidal flux tube rising under magnetic buoyancy (see Fig. 4 of Choudhuri 2003). If d is the depth of the convection zone, the flux dragged by the tube is

$$F \approx B_P d \quad (2)$$

This flux gets wrapped around the tube of radius a. In an ideal-MHD situation, this flux *F* would be confined to a narrow sheath around the flux tube. In reality, however, we expect that the turbulence around the flux tube would make this flux *F* penetrate into the flux tube. Then the magnetic field going around the tube can be taken to be of order *F/a*. The current density associated with this field is of order F/a^2 and is along the axis of the tube. If B_T is the magnetic field inside the flux tube, then it follows from (1):

$$\alpha \approx (F / a^2) / B_T \approx B_P d a / B_T \quad (3)$$

on substituting from (2) for F. We use $B_p \approx$ 1G, the depth of the SCZ d $\approx 2\text{x}10^8$ m and the field inside sunspots $B_T \approx$ 3000 G. On taking the radius of the sunspot a $\approx$ 2000 km and a $\approx$ 5000 km, we get $\alpha \approx 2\text{x}10^{-8}$ m^{-1} and $\alpha \approx 3\text{x}10^{-9}$ m^{-1} respectively. Thus, from very simple arguments, we get the correct order of magnitude.

2.1. Results from dynamo simulation

In section 4 of Chatterjee et al. (2004) we have presented a particular dynamo model, which we refer to as our standard model. We now present helicity calculations based on this standard model. A flux eruption takes place in our model whenever the toroidal field at the bottom of the SCZ exceeds a critical value. Whenever an eruption takes place in our dynamo simulation, we calculate the poloidal flux *F* through SCZ at the eruption latitude by integrating B_θ from the bottom of SCZ ($r = R_b$) to the top ($r = R_s$), i.e. $F = \int_{R_b}^{R_s} B_\theta dr$ If the sign of *F* is opposite to the sign of the toroidal field *B* at the bottom of SCZ, then the helicity is taken as negative (otherwise it is positive).

Fig.1a is a plot of helicity associated with eruptions at different latitudes. This is the theoretical plot that has to be compared with observational plots like Fig. 2 of Canfield and Pevtsov (2000). To see the variation of helicity with the cycle, Fig.1b and Fig.1c present plots of helicity for eruptions during 4 years of solar maximum and 4 years at the beginning of the cycle respectively. The straight lines represent the least-square fits.

3. Distribution of helicity in active regions and accretion of poloidal field

If the magnetic flux in the rising flux tube is nearly frozen, then we expect that the poloidal flux collected by it during its rise through the SCZ would be confined in a narrow sheath at its outer periphery. In order to produce a twist in the flux tube, the poloidal field needs to diffuse from the sheath into the tube by turbulent diffusion. However, turbulent diffusion is strongly suppressed by the magnetic field in the tube. This nonlinear diffusion process was studied in an untwisted flux tube by (Petrovay and Moreno-Insertis, 1997) who concluded that a substantial amount of flux may be eroded away from a rising flux tube during the process of its rise through the SCZ. In the present paper we extend this model by including the poloidal component of the magnetic field (i.e. the field which gets wrapped around the flux tube) and study the evolution of the magnetic field in the rising flux tube, as it keeps collecting more poloidal flux during its rise and as turbulent diffusion keeps acting on it.

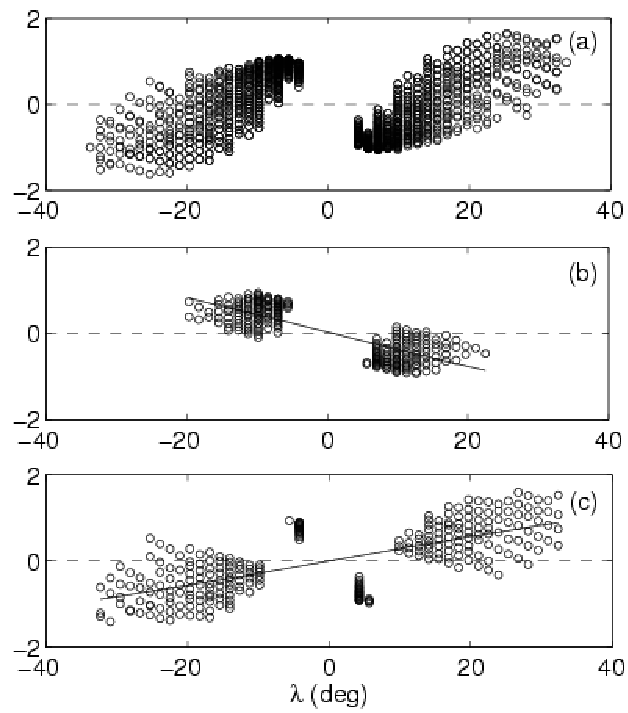

Fig. 1. Helicity α (plotted along the vertical axis in arbitrary units) for eruptions at different latitudes, denoted by open circles: (a) for an entire solar cycle; (b) for 4 years during the maximum and the declining phases of the solar cycle; (c) for the first 4 years of the solar cycle. The solid lines in (b) and (c) are least--square fits to the model results.

3.1 Equations in the comoving Lagrangian frame

Suppose we formulate our problem in a frame of reference fixed with the centre of the rising flux tube. As the flux tube rises to regions of lower density and expands, it would appear in that frame there is a radial expansion of the material inside the flux tube.

Let us assume that the material inside the flux tube expands in a self-similar fashion and use the Lagrangian position coordinate ξ instead of Eulerian coordinate *r*. Then we can write

$$\xi = F(t) r \quad (4)$$

Where *F(t)* will have to be a monotonically decreasing function of *t* for an expanding flux tube. With some simple algebra we find

$$B_z(r,\ t) = B'_z(\xi,\ t)\ F(t)^2, \tag{5}$$

$$B_\varphi(r,\ t) = B'_\varphi(\xi,\ t) F(t), \tag{6}$$

$$\frac{\partial B'_z}{\partial t} = F^2 \frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\eta\xi\frac{\partial B'_z}{\partial \xi}\right), \tag{7}$$

$$\frac{\partial B_\varphi'}{\partial t} = F^2 \frac{\partial}{\partial \xi}\left[\eta\frac{1}{\xi}\frac{\partial}{\partial \xi}(\xi B_\varphi')\right] . - F\frac{\partial}{\partial \xi}(vB\varphi) \tag{8}$$

We incorporate the accretion of poloidal flux to the tubes by assuming that the poloidal flux is brought uniformly from all directions by a radial inward flow with velocity v equal to the velocity with which the fluid is flowing from the upward direction. As the flux tube rises through the SCZ, we denote its radial distance from the centre of the Sun by *R*. The flux tube begins from the bottom of SCZ at $R = R_b$, where its radius is ξ_{ft} and the external density is ρ_{e0}. When it rises to R where the external density is ρ_e, its radius becomes r_{ft}. Since the density inside the flux tube would be very nearly equal to the external density, mass conservation implies

$$R_b \rho_{e,0} \xi_{ft}^2 = R \rho_e r_{ft}^2.$$

From (4) it follows that

$$F = \frac{\xi_{ft}}{r_{ft}} = \sqrt{\frac{R\rho_e}{R_b \rho_{e,0}}} \tag{9}$$

Thus, to find F as a function of time, we need to find out how *R* changes as a function of time and we also require a model of the SCZ which will give us the value of ρ_e at that value of *R*.

We take η to be given by the expression

$$\eta = \eta_0 / \left(1 + (B/B_{eq})^\kappa\right), \tag{10}$$

where $B = \sqrt{B_z^2 + B_\varphi^2}$ is the amplitude of the magnetic field and B_{eq} is the equipartition magnetic field. We use the convection zone model of (Unno, Kondo and Xion, 1985) to obtain B_{eq} at different positions R within SCZ.

3.2 Results

The presence of 3000 G magnetic fields in sunspots is a compelling proof that magnetic fields may never fall to such low values inside fluxtubes near the surface; in fact, at least in photospheric layers, they remain well above the equipartition value (the magnetic field inside a typical sunspot being about thrice the equipartition field). We present some calculations by artificially not allowing the magnetic field to fall below the equipartition value. Suppose the magnetic field in the interior of the flux tube falls to a value sB_{eq} at some depth (s being a numerical factor of the order of unity). We assume that the magnetic field inside the flux tube will have the value sB_{eq} in the higher layers as it rises further. If this is the case, then magnetic buoyancy would be given by

$$\frac{\rho_e - \rho_i}{2\rho_e} = \frac{s^2 Beq^2}{16\pi p_e}.$$

While we calculate the rise of the flux tube by using this expression of magnetic buoyancy, we cannot allow the cross-section to expand indefinitely if the magnetic field has to remain sB_{eq}. Instead of equation (8), we calculate F(t) by using the relation

$$F(t) = \sqrt{sB_{eq} / B_0} \tag{11}$$

Where $B_0 = 10^5$ G. Results for s=1 (case B1) and s=3 (case B3) are shown in Fig. 2 and 3 respectively. We plot B_z', B_φ' and α_p as functions of ξ at depths 0.7 R_s, 0.85R_s, 0.9 R_s, 0.95 R_s and 0.98 Rs. The times taken to reach these depths are given in the figure captions. The diffusion remains significantly quenched if the magnetic field stays higher than B_{eq} and has also less time to act because the flux tube rises fast. As a result, we see that the effect of diffusion is somewhat less for case B1 and drastically less for case B3. We see in Fig. 3 that B_z' has not diffused much and B_φ' has remained confined in a narrow sheath at the boundary of the flux tube, being unable to diffuse inward unlike in Fig. 2.

4. Conclusions

Two clear theoretical predictions follow from our helicity model. Since the helicity goes as a^{-2} as seen from (3), the smaller sunspots should statistically have stronger helicity (i.e., higher values of twist α). At the beginning of a cycle, helicity should be opposite of what is usually observed.

The results presented above indicate that the contribution of poloidal field accretion to the development of twist can be quite significant, and under favourable circumstances it can potentially account for most of the current helicity observed in active regions.

One rather strong prediction of our model is the existence of a ring of reverse current helicity on the periphery of active regions. The amplitude of the resulting twist (as measured by the mean current helicity in the inner parts of the active region) depends sensitively on the assumed structure (diffuse vs. concentrated/intermittent) of the active region magnetic field right before its emergence, and on the assumed vertical profile of the poloidal field.

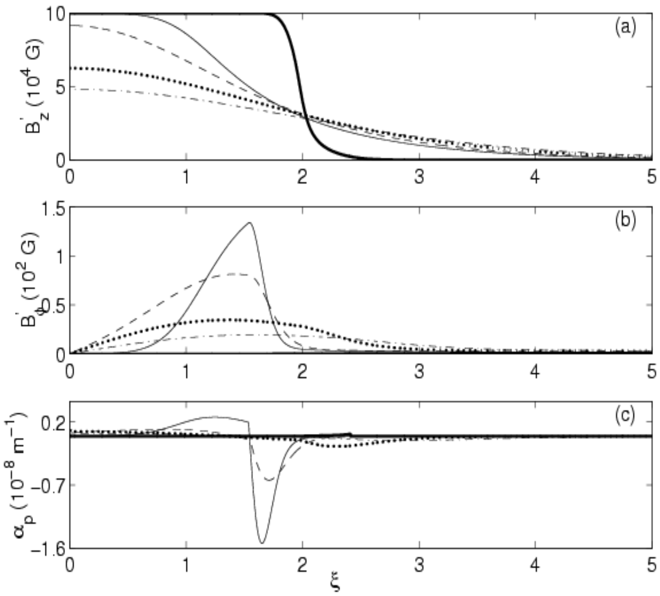

Fig. 2. Plots of B_z', B_φ' and α_p as functions of ξ for arising flux tube. The field inside the tube is not allowed to decrease below B_{eq} (case B1). The different curves correspond to the profiles of these quantities at the following positions of the flux tube: 0.7 R_s (thick solid), 0.85 R_s (solid), 0.9R_s (dashed), 0.95 R_s (dotted), 0.98 R_s (dash-dotted). The flux tube reaches these positions at times 0 days, 5.2 days, 6.6 days, 7.7 days and 8.2 days after the initial start. The values of $\left(B / B_{eq}\right)$ at the centres of these flux tubes at these positions are 10, 1.72, 1.0, 1.0, 1.0 respectively.

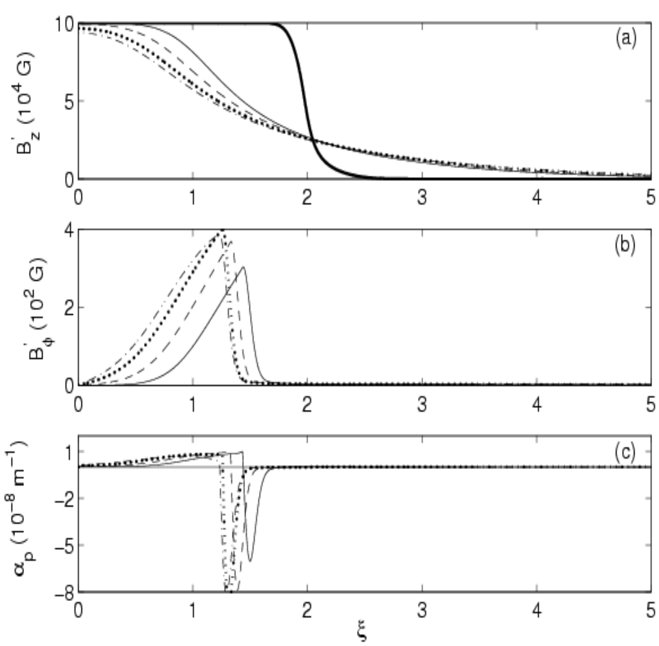

Fig. 3. Plots of B_z, B'_ϕ and α_p as functions of ξ for a rising flux tube. The field inside the tube is not allowed to decrease below B_{eq} (case B1). The different curves correspond to the profiles of these quantities at the following positions of the flux tube: 0.7 Rs (thick solid), 0.85 R_s (solid), 0.9R_s (dashed), 0.95R_s (dotted), 0.98 R_s (dash-dotted). The flux tube reaches these positions at times 0 days, 5.2 days, 6.2 days, 6.8 days and 7.0 days after the initial start. The values of (B/B_{eq}) at the centres of these flux tubes at these positions are 10, 3.0, 3.0, 3.0, 3.0 respectively.

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